Cost-Effectiveness of Pooled Spares in the Deep Space Network

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Providing a common pool of spares for types of equipment that are in operation in more than one station of a complex can result in a cost which is substantially less than the total cost of sparing each station separately. This report presents a cost-effective method of determining such spares complements, which is an extension of a previously proposed method. The generalized algorithm of the present method can also be used for unpooled sparing whenever appropriaté. Several practical examples are given which illustrate the cost savings that can be achieved by pooling spares. For these examples, savings range from 45 to 75%.

I. Introduction

A cost-effective method of spares provisioning for repairable equipment (modules) at Deep Space Stations (DSSs) is described in Ref. 1. That report considered the determination of spares packages for a system consisting of k types of modules. For the system to be operational, at least m_j modules of type j must be unfailed for $j = 1, \cdots, k$, and there are $n_j \geq m_j$ modules of type j in the system, not counting spares. The time required to replace a failed module by a spare is assumed to be negligible in this analysis. The uptime ratio (UTR) for the jth type of module is defined as the fraction of time in the long run that at least m_j modules of type j are unfailed, which depends on the number of spares, N_j, of type j. It is assumed that failures and repairs of different module types are independent, so that the system UTR is the product of the UTRs of the k types. These k UTRs are computed using the Markovian model for failures and repairs described in Ref. 2. The algorithm of Ref. 1 is then used to determine spares packages $(N_1, \cdots, N_k)$ which are cost-effective in that they yield system UTRs which cannot be improved upon without increasing the total cost.

The present report describes the extension of this method, which was developed to deal with actual sparing problems in the DSN involving an additional feature: the possibility of pooling spares for three stations at the same complex, with different system configurations. Examples of the cost-effective spares packages developed for these problems are presented in Section III. Comparisons with the cost of unpooled spares packages for the individual stations are given. It is shown that to achieve the same station UTRs, the cost of pooled spares is substantially smaller.

II. Extension of the Sparing Algorithm

Whenever a module fails at the ith station ($i = 1,\cdots,s$), it is replaced by a spare of the same type, unless there are none in the pool, in which case a "back order" is recorded. In the latter case, it is assumed that when the repaired modules of that type become available, the back orders are filled in the same order as they were recorded. One could do better sometimes by filling back orders judiciously; for example, by rescuing a station from a down condition whenever possible. But this would require monitoring the numbers of operating modules at the stations and would not result in significant improvements.

Under these assumptions, the stationary (long-term) probabilities of $0,\cdots,n_j$ modules of type j operating at, say, station 1 can be obtained as follows. First, use the algorithm of Ref. 2 to find the stationary probabilities $p_0,p_1,\cdots,p_{N_j+r_j}$ of $0,\cdots,N_j+r_j$ modules being failed in the entire complex, where N_j = total number of spares of type j, and r_j = total number operating in all the stations. If the number failed is V, say, and V is N_j or less, then all n_j modules will be operating at station 1. If $V > N_j$, however, then $V-N_j$ modules among the r_j are down. The probability that exactly i of these are at station 1 is

$$\frac{\binom{n_j}{i}\binom{r_j-n_j}{V-N_j-i}}{\binom{r_j}{V-N_j}}$$

where $\max(0,V-N_j+n_j-r_j) \leq i \leq \min(n_j,V-N_j)$. Hence, the stationary probability that exactly i ($i > 0$) of the type j modules are down at station 1 is

$$\sum_{V=i+N_j}^{i+N_j+r_j-n_j} p_V \frac{\binom{n_j}{i}\binom{r_j-n_j}{V-N_j-i}}{\binom{r_j}{V-N_j}}$$

Summing over $i > n_j-m_j$ gives the stationary probability of the jth module type being down at station 1. Subtracting this sum from one gives the UTR of the jth module type at station 1. Multiplying the results over $j = 1,\cdots,k$ yields the station UTR for station 1. The cost-effective spares provisioning algorithm can then be applied with the "value" (see Ref. 1) of a spares package defined to be the sum of the log UTRs of the individual stations (thus weighting all stations equally).

III. Examples of Cost Savings Through Spares Pooling

The first example is concerned with terminet sparing at Goldstone. A spares complement was to be provided for a total of 552 operating modules, 24 modules of each of 23 types. Three cases were considered. For the first case it was assumed that all the modules were contained in one system and that for each type of module $m_j=n_j = 24$. In effect, this means that if any of the 552 operating modules fails and there is no spare available to replace it, the system is down. For the remaining cases it was assumed that there are three identical systems, one at each station and operating independently, and that for each system $m_j=n_j = 8$ for each of the types. However, for the second case a common pool of spares was to be provided for these three systems, while in the third case each system was to have its own spares complement. In all cases, spares packages were generated using the extended algorithm as given in Section II, which can also be used for the unpooled case when the proper parameter values are inserted. Repair time was assumed to be two weeks. Table 1 gives the pertinent results for spares packages with UTRs of about 0.90, 0.95 and 0.99. The UTRs shown for the first case are the fractions of time that the total system is operational, while the UTRs shown for the second and third cases are those for each station. The contents of the spares packages are omitted. It can be seen from the costs shown in Table 1 that pooled sparing in this example results in a cost savings of about 55% for comparable UTRs.

The next example is concerned with magnetic tape sparing at Goldstone. There are three independent, identical systems, one at each station. Each system is made up of 2 modules each of 5 types and 4 modules of a sixth type, a total of 42 operating modules. Four cases are considered. For cases 1 and 2, we assume for each system that $m_j=n_j = 2$ for $j = 1,\cdots,5$ and $m_6=n_6 = 4$. For cases 3 and 4, we assume that the n_j's are the same as in cases 1 and 2 but that for each system $m_j = 1, j = 1,\cdots,5$

and $m_6 = 2$. For cases 1 and 3, we assume pooled sparing, and for cases 2 and 4 separate spares complements for the three systems. Repair time was assumed to be 6 weeks. Table 2 gives some of the results obtained. A cost savings of about 45% is achieved by pooling spares. This example also illustrates the importance of the m_j's, since their values relative to those of the respective n_j's reflect the amount of redundancy in the system. This explains the high UTR achieved for cases 3 and 4 when no spares are provided.

The final example concerns megadata terminal sparing at JPL and involves an unusual situation. There are seven independent systems. System 1 consists of 2 each of 9 types of modules, while systems 2 through 7 each consist of 1 each of the same 9 types. Thus, any spares pool for the seven systems will result in a UTR_1 for system 1 and a UTR_2 for each of the remaining systems, where $UTR_1 \neq UTR_2$. We consider four cases. For cases 1 and 2, we assume that for each system $m_j = n_j$ for all j. For cases 3 and 4, we assume that for system 1, $m_j = 1$ for all j. For cases 1 and 3, we assume pooled sparing, while for cases 2 and 4 separate sparing. Table 3 lists some of these results obtained. A comparison between the pooled and separate sparing shows an average cost savings of about 75%. This example illustrates the fact that, in general, as the number of systems containing the same types of modules increases, the cost savings for pooled sparing, rather than separate sparing, also increase.

IV. Conclusions

The cost of pooling spares for several systems containing the same types of modules will always be less than the total cost of sparing each system separately. The extended algorithm, designed for pooled sparing, can be applied to systems containing redundant as well as nonredundant elements. Thus, this algorithm provides a flexible method for achieving substantial reductions in sparing costs.

References

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2. Eisenberger, I., Lorden, G., and Maiocco, F., "A Preliminary Study of Spares Provisioning for the Deep Space Network," in *The Deep Space Network Progress Report*, Technical Report 32-1526, Vol. XVIII, pp. 102–110, Jet Propulsion Laboratory, Pasadena, Calif., Dec. 15, 1973.

Table 1. Terminet sparing at Goldstone (23 types of modules, 24 modules of each type)

One system			Three systems, pooled sparing			Three systems, separate sparing		
Number of spares	System UTR	Spares cost, $	Number of spares	System UTR	Spares cost, $	Number of spares	System UTR	Spares cost, $
0	0.038	0	0	0.335	0	0	0.335	0
31	0.905	3499	23	0.915	2608	57	0.901	5808
38	0.952	4155	27	0.952	3190	66	0.951	7086
48	0.990	5337	40	0.990	4576	93	0.990	10497

Table 2. Magnetic tape sparing at Goldstone (6 types of modules; 6 modules each of 5 types; 12 modules of sixth type)

Pooled sparing $m_j = n_j$			Separate sparing $m_j = n_j$			Pooled sparing $m_j < n_j$			Separate sparing $m_j < n_j$		
Number of spares	System UTR	Spares cost, $	Number of spares	System UTR	Spares cost, $	Number of spares	System UTR	Spares cost, $	Number of spares	System UTR	Spares cost, $
0	0.363	0	0	0.363	0	0	0.9792	0	0	0.9792	0
10	0.944	30480	21	0.946	53820	4	0.9924	8770	3	0.9925	17310
13	0.981	38250	24	0.982	71130	5	0.9967	14540	12	0.9964	26310
14	0.991	44020	33	0.990	80130	9	0.9993	28080	15	0.9990	43620

Table 3. Megadata terminal sparing at JPL (9 types of modules; one system of 2 modules of each type; 6 systems each containing 1 module of each type)

Case 1 Pooled sparing				Case 2 Separate sparing				Case 3 Pooled sparing				Case 4 Separate sparing			
Number of spares	System UTR_1	System UTR_2	Spares cost, $	Number of spares	System UTR_1	System UTR_2	Spares cost, $	Number of spares	System UTR_1	System UTR_2	Spares cost, $	Number of spares	System UTR_1	System UTR_2	Spares cost, $
0	0.585	0.765	0	0	0.585	0.765	0	0	0.9807	0.765	0	0	0.9807	0.765	0
8	0.902	0.949	4588	36	0.907	0.951	20098	4	0.9974	0.904	2888	14	0.9969	0.902	9660
12	0.979	0.989	8245	64	0.976	0.989	39275	8	0.9983	0.949	4588	33	0.9984	0.951	19470
15	0.992	0.996	10453	71	0.994	0.998	48235	12	0.9996	0.989	9445	62	0.9996	0.989	38098